

OCTOBER 06 2025

A refined bow-string interaction model considering hysteresis

Ewa Matusiak; Vasileios Chatziioannou ; Maarten van Walstijn



Proc. Mtgs. Acoust. 58, 035014 (2025)

<https://doi.org/10.1121/2.0002120>



Articles You May Be Interested In

Studies in bowing point friction in bowed strings

J. Acoust. Soc. Am. (May 1998)

Effect of single and multiple acoustic frequencies on the atomization of a spray

Physics of Fluids (December 2024)

Friction force reconstruction in bowed cello strings

J. Acoust. Soc. Am. (April 2025)



LEARN MORE

Advance your science and career as a member of the
Acoustical Society of America



International Symposium on Music and Room Acoustics

24-28 May 2025

Loyola University
New Orleans, Louisiana

Musical Acoustics

A refined bow-string interaction model considering hysteresis

Ewa Matusiak

Department of Music Acoustics (IWK), University of Music and Performing Arts Vienna: Universitat fur Musik und darstellende Kunst Wien, Vienna, 1030, AUSTRIA; matusiak@mdw.ac.at

Vasileios Chatziioannou

University of Music and Performing Arts Vienna: Universitat fur Musik und darstellende Kunst Wien Vienna, AUSTRIA; chatziioannou@mdw.ac.at

Maarten van Walstijn

Queen's University Belfast, Belfast, UNITED KINGDOM; m.vanwalstijn@qub.ac.uk

In order to accurately model the bow-string interaction, a friction model is necessary that will correctly predict the friction force during stick-slip phases. This force has been estimated from experimental measurements. It appears to be at its highest at the start of slipping, and it decreases as the slipping speed increases. On the way back towards the sticking phase, the force is lower than on the way towards the slipping phase. It has been hypothesized that the temperature fluctuations of rosin are responsible for this hysteretic behaviour [Smith, J. and Woodhouse, J., The tribology of rosin. *J. Mech. Phys. Solids*, 48(8), 2000]. In this work, a model is proposed that directly incorporates the temperature changes of rosin during bowing. It is shown that the friction force resembles the one estimated from the measurements.

1. INTRODUCTION

The bowed string and especially the non-linear phenomena occurring at the bow-string contact has been a subject of numerous studies.^{4,8–11} Particularly challenging is the modelling of stick-slip behaviour occurring during bowing. Numerous friction models based on observations have been suggested^{3,14,16} but none can accurately predict the string's different regimes of oscillations from only bow force, acceleration and bowing position. It has been long assumed that the only parameters shaping friction force are the relative bow-string velocity and the bowing force. Reconstruction of friction force from experimental measurements^{12,17} indicates that the friction force is highest at the start of slipping, then it falls as the slipping speed increases, and on the way back towards the sticking phase, the force remains much lower than the maximum value on the way out. That results in a hysteresis loop in the friction force versus velocity plane, indicating that relative velocity can not be the only parameter shaping friction force. It has been suggested¹⁴ that the variable which governs the friction force is not only the sliding speed, but the rosin temperature in the contact region needs to be taken into account. Since the contacting bodies have some thermal inertia, changes in temperature lag behind changes in sliding speed, producing a hysteresis loop. Rosin is a glassy material that transitions from a solid to a liquid state depending on the temperature it is exposed to. As a lubricant, it was first studied in Ref. 1 and later examined in Ref. 14 in the context of bowed string instruments giving rise to a thermal friction model. The thermal model explains the falling friction-velocity characteristic, measured during steady sliding experiments, in terms of melt lubrication. As the sliding velocity increases, the temperature of the rosin layer rises and its viscosity drops resulting in a reduced resistance to sliding. This description gives a physical explanation/interpretation for the hysteresis observed measurements of the stick-slip friction-velocity characteristics of rosin coated surfaces. The accuracy of the thermal model to correctly predict string oscillations was assessed by comparing a simulated Guettler diagram with an experimentally obtained one in Ref. 4. The thermal friction¹⁴ model performed better than any velocity-only-dependent friction paradigm recovering roughly the playability region. It was able to simulate various steady-state regimes but reconstructing the transient part of bowed strings still posed a challenge.

In the last two decades an asperity based elasto-plastic friction model has been used in bow-string simulations. The elasto-plastic model was developed in Ref. 2 and utilized for simulating bowed-strings first in Refs. 13, 15 and later in Refs. 6, 7 where it was refined to be guaranteed-passive⁷ enabling stable simulations. This friction model assumes that the two surfaces, the bow and the string, are irregular at the microscopic level, and their contact is modelled through an ensemble of elastic bristles, each contributing to the total friction load. As the bow drives the string, the bristles stretch to the point of breaking and the string starts to slide. The model has been shown to reconstruct both the steady-state and the transient part of the signal and the global performance was compared to measurements with the help of Guettler diagrams⁶ predicting roughly the correct playability region. In previous frictions models hysteresis rules were imposed in order to impose uniqueness of the solution. In this work, since the friction force is governed by relative velocity and bristle displacement through a differential equation which possesses a unique solution, hysteresis loops arise naturally. However, the hysteresis loops differ considerably from those observed in experimentally measured waveforms.¹⁷ Hysteresis loops from the elasto-plastic friction model are confined only to lower velocities. For higher relative velocities the friction force is constant, in contrast to the experimental observations.⁶

The range of bristle displacements over which combined elastic and plastic sliding occurs in the elasto-plastic friction model is a function of relative velocity. Introducing an additional fluid-related state variable should provide better consistency with the underlying physics. Therefore, the changing viscosity of rosin (due to temperature fluctuations) as an elasto-plastic lubricant should be reflected in the bristle breaking point. In this article, a temperature-dependent asperity-based friction model is suggested that regulates the bristle displacement and the break-away point according to the viscosity of rosin, effectively merging thermal and elasto-plastic friction models.

In Section 2 the elasto-plastic model is briefly recounted and a refinement taking into account changes in the rosin temperature. The application of a new model to bowed strings is described in Section 3 followed by a brief description of a discretization scheme. Finally in Section 4 the performance of the model is presented via an example and the refined model is compared to the original temperature-free elasto-plastic model.

2. PHYSICAL MODEL

The elasto-plastic friction model is an asperity-based friction model developed first in Ref.2. This model assumes that the two surfaces, in this case the bow hair and the string, are irregular at the microscopic level, and their contact is modelled through an ensemble of elastic bristles, each contributing to the total friction load, see Fig. 1. The bristles are modelled as damped stiff springs and when the strain exceeds a certain break-away threshold, the bristles break, and the two surfaces begin to slide.

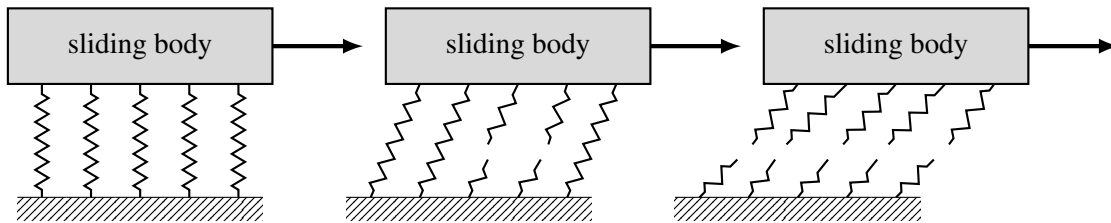


Figure 1: Bristle interaction between the two sliding bodies.

Denoting by z the average bristle deflection and by v the relative velocity between the string and the bow, the model is described as follows:

$$F = \sigma_0 z + \sigma_1(v) \dot{z}, \quad (1)$$

$$\dot{z} = v \left[1 - \alpha(z, v) \frac{z}{z_{ss}(v)} \right], \quad (2)$$

where σ_0 in $[\text{kg/s}^2]$ and σ_1 in $[\text{kg/s}]$ are bristle stiffness and damping, respectively. The change of rate with which the bristles stretch or contract is related to the relative velocity through the adhesion map α , defined as

$$\alpha(z, v) = \begin{cases} 0, & vz \leq 0 \\ \alpha_m(z, v), & vz > 0, \end{cases} \quad (3)$$

where

$$\alpha_m(z, v) = \begin{cases} 0, & |z| \leq z_{ba} \\ \bar{\alpha}_m(v, z), & z_{ba} < |z| < |z_{ss}(v)| \\ 1, & |z| \geq |z_{ss}(v)|, \end{cases} \quad (4)$$

where $z_{ba} \leq \mu_C f_N / \sigma_0$ is the break-away displacement,

$$\bar{\alpha}_m(z, v) = \frac{1}{2} \left[\sin(\pi \theta(z, v)) + 1 \right], \quad \theta(z, v) = \frac{|z| - \frac{1}{2}(|z_{ss}(v)| + z_{ba})}{|z_{ss}(v)| - z_{ba}}, \quad (5)$$

and z_{ss} is the steady-state displacement for constant velocities

$$z_{ss}(v) = \begin{cases} \frac{f_N}{\sigma_0} \left(\mu_C + (\mu_S - \mu_C) e^{-|v/v_S|^p} \right), & v \geq 0 \\ -\frac{f_N}{\sigma_0} \left(\mu_C + (\mu_S - \mu_C) e^{-|v/v_S|^p} \right), & v < 0, \end{cases} \quad (6)$$

with $v_S > 0$ and $p \geq 1$. When $p = 2$, v_S is referred to as Stribeck velocity. Here, μ_S and μ_C denote static and dynamic friction coefficients, respectively, and f_N is the normal force applied by the bow. For small bristle displacements, when $|z| \leq z_{ba}$, $\alpha(v, z) = 0$ and consequently $\dot{z} = v$, a purely elastic and reversible regime is entered, referred to as pre-sliding (sticking). For larger displacements, that is when $z_{ba} < |z| < |z_{ss}(v)|$, some bristles start to break and a mixed elasto-plastic sliding occurs. Finally, for $|z| \geq |z_{ss}(v)|$ all bristles break, and a purely plastic regime is achieved, that is the string slips under the bow. In that situation, $\alpha(z, v) = 1$. The original elasto-plastic model assumed that the bristle damping is constant. It was shown in Ref. 7 that the model is guaranteed-passive when the bristle damping varies with relative velocity v and normal force f_N , as

$$\sigma_1(v) = \frac{\mu_C f_N}{\sqrt{v^2 + \epsilon^2}}, \quad \epsilon = \frac{\mu_C f_N}{\bar{\sigma}_1}, \quad (7)$$

such that $0 \leq \sigma_1(v) \leq \bar{\sigma}_1$ and $\sigma_1(0) = \bar{\sigma}_1$.

The rosin is a material that changes its state due to temperature, changing effectively the viscosity of the rosin. That in turn is reflected in the steady-state displacement function, the Stribeck curve. In order to incorporate the influence of temperature in the elasto-plastic model, the Stribeck curve is adapted according to the changes in viscosity. The variation of viscosity with the temperature was reported in Ref. 14 and was observed to follow an exponential trend, well approximated by

$$\eta(T) = \eta_0 \cdot \tilde{\eta}(T), \quad \tilde{\eta}(T) = 10^{\frac{20-T}{40}}, \quad (8)$$

where T is the temperature in Celsius and η_0 is the viscosity of rosin at room temperature $T_0 = 20^\circ\text{C}$, see Fig. 2 (plot on the right). Let

$$\tilde{\sigma}_0(T) = \sigma_0 \tilde{\eta}(T)^{-1}, \quad (9)$$

such that $\tilde{\sigma}_0(T_0) = \sigma_0$, and define the underlying Stribeck curve of the elasto-plastic model, that is the steady-state displacement, as a function of v and T

$$z_{ss}(v, T) = \begin{cases} \frac{f_N}{\tilde{\sigma}_0(T)} \left(\mu_C + (\mu_S - \mu_C) e^{-|v/v_S|^p} \right), & v \geq 0 \\ -\frac{f_N}{\tilde{\sigma}_0(T)} \left(\mu_C + (\mu_S - \mu_C) e^{-|v/v_S|^p} \right), & v < 0. \end{cases} \quad (10)$$

The adhesion map, eq. (3), changes accordingly

$$\alpha(z, v, T) = \begin{cases} 0, & vz \leq 0 \\ \alpha_m(z, v, T), & vz > 0, \end{cases} \quad (11)$$

where

$$\alpha_m(z, v, T) = \begin{cases} 0, & |z| \leq z_{ba}(T) \\ \bar{\alpha}_m(v, z, T), & z_{ba}(T) < |z| < |z_{ss}(v, T)| \\ 1, & |z| \geq |z_{ss}(v, T)|, \end{cases} \quad (12)$$

with $z_{ba}(T) \leq \mu_C f_N / \tilde{\sigma}_0(T)$ the break-away displacement, and

$$\bar{\alpha}_m(z, v, T) = \frac{1}{2} \left[\sin(\pi \theta(z, v, T)) + 1 \right], \quad \theta(z, v, T) = \frac{|z| - \frac{1}{2}(|z_{ss}(v, T)| + z_{ba}(T))}{|z_{ss}(v, T)| - z_{ba}(T)}. \quad (13)$$

Lower viscosity of a lubricant results in lower friction. That is, for low viscosities the bristles start to break much earlier, triggering slipping. The steady-state displacement functions for different values of

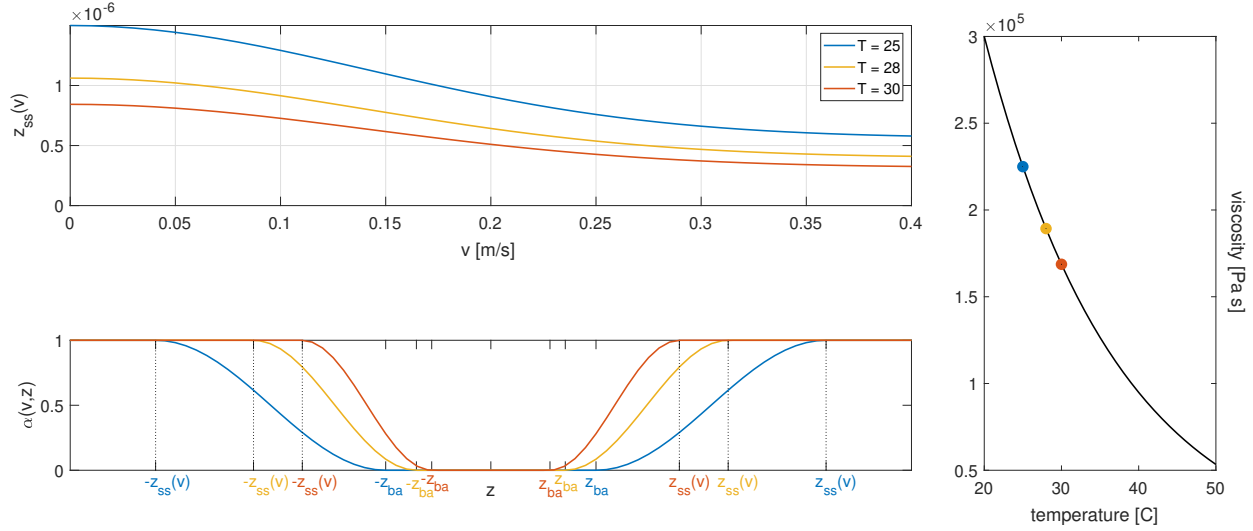


Figure 2: Top: steady-state bristle displacement functions for different temperature values. Bottom: a curve modelling bristle displacement z according to the relative velocity of the string with respect to the bow, v , for different steady state functions. When z is between $-z_{ba}$ and z_{ba} the string sticks to the bow, in the regions $(-z_{ss}(v), -z_{ba})$ and $(z_{ba}, z_{ss}(v))$ the bristles start to break away (elastic displacement) and when $|z| \geq |z_{ss}(v)|$ the model enters a plastic phase, and the string slides. For higher temperatures, the bristles start to break away sooner. Right: viscosity as a function of temperature according to eq. (8) with $\eta_0 = 3 \cdot 10^5$.

temperature are plotted in Fig. 2. They result in different shapes of the adhesion map which rules the change of rate with which the bristles stretch or contract.

For the model to include the lubricant's viscosity, the contact temperature has to be monitored. A simplified equation of rate of change of temperature is proposed here where it is assumed that thermal conductivity κ_r in [W/m/K] does not change significantly as temperature is varied. Additionally, it is assumed that the sliding surfaces (the bow and the string) have a constant temperature T_0 and that the heat diffusion rates are determined only by the properties of rosin. Then, the rate of change of the rosin temperature can be expressed as

$$A\delta\rho_r c_r \dot{T} = -\kappa_r \mathcal{W}_b (T - T_0) + Fv, \quad (14)$$

where ρ_r in [kg/m³] is the mass density of rosin and c_r in [J/kg/K] is the specific heat capacity of rosin. The layer of rosin is δ thick and the area of contact is A , while $\mathcal{W}_b$ is the bow width. The first term on the right hand side of eq. (14) describes the heat transfer from rosin layer to the surfaces of the string and the bow, while the second term is the rate of heat generation coming from friction.

The equation governing contact temperature, eq. (14), differs slightly from the one suggested by Smith and Woodhouse in Ref. 14. There the authors shaped the contact temperature as

$$A\delta\rho_r c_r \dot{T} = - \int_{-\infty}^t g(t - \tau) T(\tau) d\tau - \mathcal{W}_b \delta \rho_r c_r v T + Fv, \quad (15)$$

modelling the heat transfer with the help of a suitable Green's function for the heat diffusion problem and adding the convection with the rosin (second term on the right hand side of eq. (15)).

3. BOWED STRING

A bowed string model that omits body coupling and simplifies string damping is chosen for the demonstration of the results. This simplified approach is favoured in this case, as including these additional factors would not enhance the presentation of the friction model.

The governing equations for the motion of the string excited by a bow are the equations describing transverse (eq. (16)) and torsional (eq. (17)) waves. They are coupled through the distributed friction force f ([N/m]) and this force in turn is linked to the bow hair displacement η (see eq. (18)). The bow hair is modelled as a harmonic oscillator.¹⁰ The friction force is modelled according to the elasto-plastic friction model enhanced with temperature dependency. The partial differential equations describing the motion of the bowed string are,^{10,11}

$$\rho A \partial_t^2 u = T \partial_x^2 u - EI \partial_x^4 u - 2\gamma_0 \rho A \partial_t u + 2\gamma_1 \rho A \partial_t \partial_x^2 u - f, \quad (16)$$

$$P_T \partial_t^2 w = K_T \partial_x^2 w - 2\gamma_2 P_T \partial_t w + r f, \quad (17)$$

$$D_h \partial_t^2 \eta = -K_h \eta - \Gamma_h \partial_t \eta - f, \quad (18)$$

where ρ is the material density, $A = \pi r^2$ the cross-sectional area of the string with radius r , T is the tension of the string, E the Young's modulus, $I = \pi r^4/4$ the area moment of inertia, and γ_0 and γ_1 represent frequency independent and frequency dependent damping. Moreover, P_T denotes the polar moment of inertia, K_T torsional stiffness and γ_2 is a torsional damping coefficient. The bow stick is regarded as a rigid frame moving at a given velocity v_b and supporting a ribbon of compliant bow-hair of density D_h ([kg/m]) with distributed spring and damping constants K_h and Γ_h , respectively. The relative bow-string velocity is then expressed as

$$v = (\partial_t u - r \partial_t w) - (v_b - \partial_t \eta). \quad (19)$$

Assuming simply supported ends, the boundary conditions for the transverse movement of the string are

$$u(x, t)|_{x=0, L} = 0, \quad \partial_x^2 u|_{x=0, L} = 0, \quad (20)$$

and for the torsional movement we assume fixed boundary conditions,

$$w(x, t)|_{x=0, L} = 0. \quad (21)$$

Assuming distributed contact, the friction force f is

$$f(z, v, T) = \sigma_0 z + \sigma_1(v) \partial_t z, \quad (22)$$

where σ_0 and σ_1 are now distributed stiffness and distributed damping respectively, and $\partial_t z$ is the time derivative of z and is related to v and T through

$$\partial_t z = v \left[1 - \alpha(z, v, T) \frac{z}{z_{ss}(v, T)} \right], \quad (23)$$

where the adhesion map α is defined in eq. (3) and $z_{ss}(v, T)$ is a steady-state displacement function given by the Stribeck curve, eq. (10). The damping term $\sigma_1(v)$ is defined in eq. (7). For the distributed contact, the equation governing the changes in temperature becomes

$$A \delta \rho_r c_r \partial_t T = -\kappa_r \mathcal{W}_b (T - T_0) + f v, \quad (24)$$

where now c_r in [J/kg/K/m]) and κ_r in [W/K/m²] are distributed specific heat capacity of rosin and thermal conductivity, respectively. The contact region and the thickness of the deforming layer of rosin are both hard to determine from the experimental measurements. The parameters used here were chosen to be within a plausible range to result in roughly similar temperature fluctuations as simulated in Ref. 14.

In order to numerically simulate the bowed string, a finite difference numerical method is used to discretize the system. The energy-conserving discretization scheme proposed in Ref. 7 is used here to discretize eq. (16) - eq. (18) and eq. (22) - eq. (23). Equation (24) is discretized using a forward difference operator,

$$\mathbf{T}^{n+1} = \Delta_t \left[-\frac{\kappa_r \mathcal{W}_b}{A \delta \rho_r c_r} (\mathbf{T}^n - T_0) + \frac{\mathbf{f}^n \odot \mathbf{v}^n}{M A \delta \rho_r c_r} \right] + \mathbf{T}^n, \quad (25)$$

where Δ_t is the time step and $\mathbf{T}^n, \mathbf{v}^n, \mathbf{f}^n$ are vectors of temperature, velocity and friction force at time step n with entries corresponding to the M points along the contact region between bow and string, and $\odot$ denotes the Hadamard product.

First the temperature $\mathbf{T}^n$ is calculated from the previous time step. Then Newton's method is used to compute the velocity and bristle displacement necessary to update the equations of motion eq. (16) - eq. (18) as in Ref. 7.

String parameters		value	Bow parameters		value
L	string length [m]	0.7	a_b	bow acceleration [m/s^2]	0.8722
r	string radius [m]	$5 \cdot 10^{-4}$	v_b	bow velocity [m/s]	0.3439
T	string tension [N]	149.74	f_N	bow force [N/m]	2.3433
ρ	material density [kg/m^3]	10128			
E	Young's modulus [Pa]	$1.37 \cdot 10^{10}$	β	bow position [-]	0.0786
c	wave speed [m/s]	137.2	$\mathcal{W}_b$	bow-hair width [m]	0.01
γ_0	freq. ind. damping [s^{-1}]	1.537			
γ_1	freq. dep. damping [m^2/s]	0.0087			
K_T	torsional stiffness [$\text{N}\cdot\text{m}^2$]	$3.03 \cdot 10^{-4}$	D_h	bow-hair density [kg/m]	0.45
P_T	polar moment of inertia [$\text{kg}\cdot\text{m}$]	$4.2 \cdot 10^{-10}$	K_h	bow-hair stiffness [N/m^2]	$4.8 \cdot 10^6$
γ_2	torsional damping [1/s]	0.0172	Γ_h	bow-hair damping [kg/s/m]	$5.7 \cdot 10^3$

Table 1: String and bow parameters used to simulate the waveform in Fig. 3 and Fig. 4.

4. NUMERICAL EXAMPLES

To demonstrate the behaviour of the model, bowing a cello D string is simulated. For these simulations, the bow starts in contact with the string, and the force is kept constant while the bow accelerates from rest with a chosen acceleration value until the steady-state bow velocity v_b is reached. The string and bowing parameters are listed in Table 1 and the sampling frequency is 50kHz. The contact area was set to $\pi r \mathcal{W}_b$ at 1N normal bow force and was tuned according to the bow force value. That parameter was set here somewhat arbitrarily as half the string circumference times bow width for a normal force of 1N, but it is sufficient for presenting the behaviour of the model.

Friction model parameters		value	Rosin parameters		value
σ_0	bristle stiffness [N/m^2]	$3.186 \cdot 10^5$	ρ_r	density [kg/m^3]	1010
σ_1	bristle damping [$\text{kg}/(\text{m}\cdot\text{s})$]	0.0027	c_r	specific heat capacity [J/kg/K/m]	$2.5 \cdot 10^4$
v_S	Stribeck velocity [m/s]	0.228	κ_r	thermal conductivity [W/K/m^2]	11.3
μ_C	dynamic friction [-]	0.5071			
μ_S	static friction [-]	1.0207			

Table 2: Parameters of elasto-plastic model and of rosin's properties.

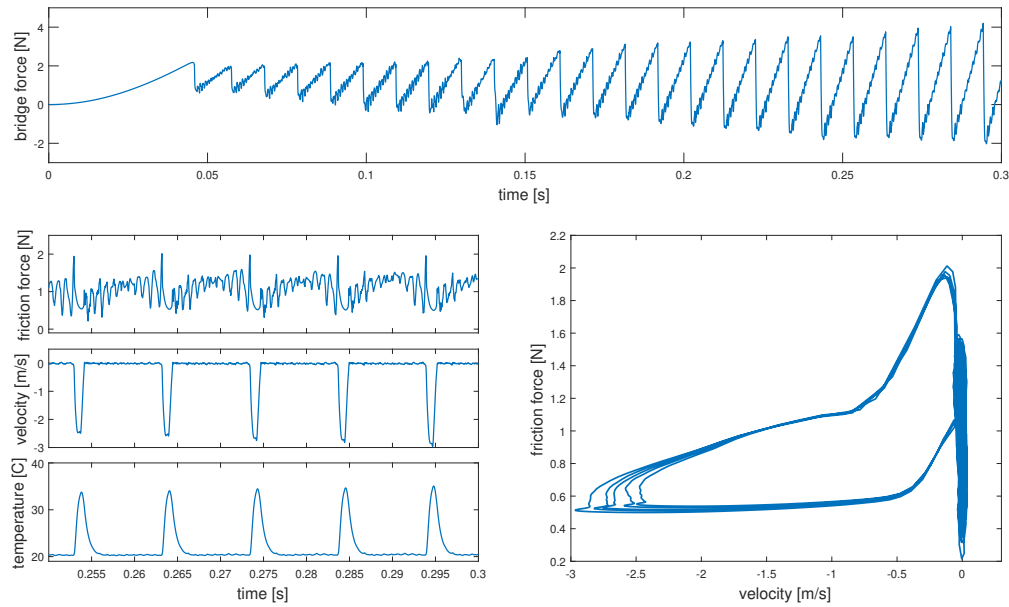


Figure 3: Helmholtz motion. Top: simulated bridge force. Bottom left: the friction force, relative velocity and corresponding temperature for the last four periods of oscillation. Bottom right: hysteresis loop of the last five periods of the simulation.

A simulated waveform is shown in Fig 3, top plot. The string exhibits Helmholtz motion from the first slip. Comparing carefully, it can be seen that the force is high at the start of slipping, and reduces during the period of slipping. The sharp peaks of friction force appear always after the onset of a slip while the peaks in temperature occur when the string starts to slow down. It can be observed that during slipping, instead of a single friction curve, there are loops. The paths around these loops move anti-clockwise. The details of the loop shapes can vary in different bow strokes, but the qualitative behaviour is always very similar, agreeing with the observations in Ref. 12. The temperature rises quickly during the slip and as the string slows down the temperature starts to drop until it reaches a minimum in the stick-phase of bowing. The temperature ranges reported here are much lower than the ones calculated in Refs. 14, 16 using the thermal friction model and eq. (15). There could be numerous reasons for that discrepancy, ranging from the model of temperature chosen here to an oversimplified impact of temperature changes on the Stribeck curve. Apart from the model choice, the factor that influences temperature fluctuations during bowing is the effective contact area A , which was chosen here rather arbitrarily since in practice it is infeasible to measure. However, there are no measurements of contact temperature to assess which ranges are correct.

As a comparison, the same parameters were used to simulate the waveform in Fig. 4 using the original elasto-plastic friction model. It can be observed that the hysteresis loops are largely confined to sliding speeds close to zero. The friction force always shows a rise followed by a marked dip, and for higher sliding speeds it traces out much the same curve for rising and falling velocities in contrast to what is observed in the reconstructed friction curves.¹⁷ Using the refined model, due to the dependency on the temperature, the friction force occupies a wider range of values, from 0.2N to 2N, while the friction force resulting from the original elasto-plastic friction model is confined to a much smaller region between 1.2N to 2.2N. In this particular example, both models predicted Helmholtz motion from the first slip with bridge force waveforms being very similar. However for different pairs of bow force and acceleration the two models can predict different transients which can be best observed by comparing respective Guettler diagrams.⁵

Guettler diagram serves as a measure in assessing the playability of a bowed string by measuring the number of periods necessary to arrive at Helmholtz motion. Such a diagram is generated by choosing a fixed

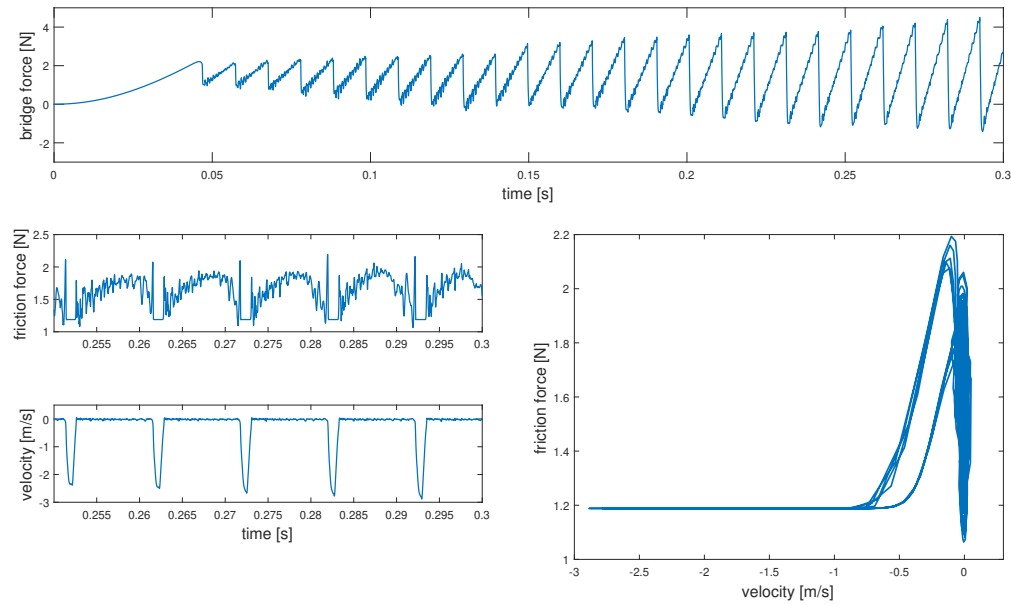


Figure 4: Helmholtz motion as in Fig.3 simulated using the original elasto-plastic friction model. The friction force is constant for a wide range of velocities during slipping.

location on the string and bowing from rest with an accelerating bow and constant bow force. The bowing is repeated for different pairs of acceleration and bow force, that are incremented in small steps. The amount of periods needed to reach Helmholtz motion is represented by the colour (white corresponds to 0 and black to 20 or more periods, with intermediate transient lengths having different shades of grey) of each pixel (see Fig. 5).

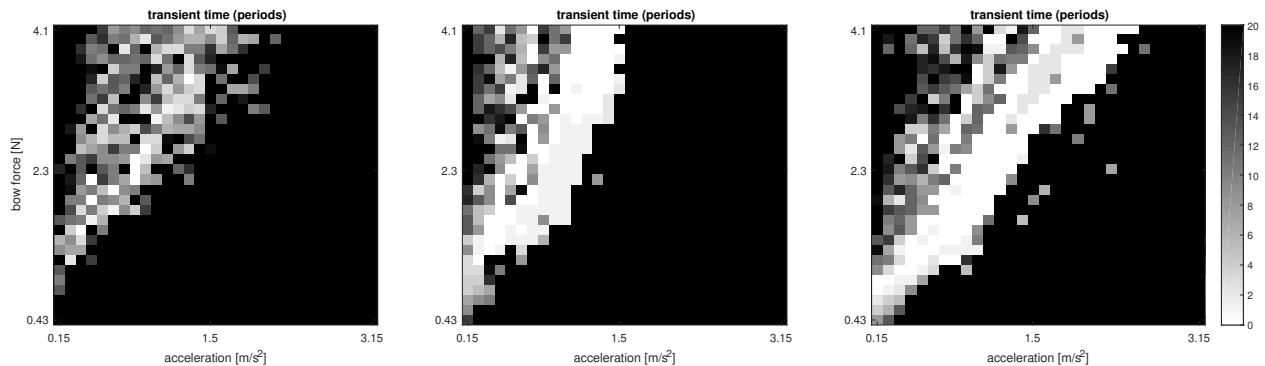


Figure 5: Guettler diagrams for cello D string obtained from experimentally bowed string (left), simulated using refined elasto-plastic friction model (middle) and simulated using thermal elasto-plastic model (right). The string was bowed at 5.5cm from the "bridge".

Three Guettler diagrams, generated for a cello D string with parameters as in Table 1, are shown for comparison in Fig. 5. The left one was experimentally obtained by a robotic arm bowing a monochord, the middle one was simulated using a refined elasto-plastic friction model, while the Guettler diagram on the right was simulated using a thermal elasto-plastic friction model. The friction parameters needed for the models are listed in Table 2. The chaotic nature of the frictional interaction is demonstrated by the "speckiness" of the playability regions: small changes in either bow force or acceleration result in very different transient times. The thermal elasto-plastic model seems to be more "playable" than a refined elasto-

plastic friction model occupying more area in the bow force-acceleration plane. Moreover the playability region of this model agrees more with the playability region of the experimentally obtained Guettler diagram.

For a more conclusive and meaningful comparison of the ability of thermal elasto-plastic model to properly predict measurements, measured Stribeck curves are necessary. In the present model, a simple scaling of the Stribeck curve according to drop in viscosity due to temperature changes was applied, however a more realistic dependency of Stribeck curve on viscosity is needed. Another question remains how to assess bristle's stiffness and damping coefficients. The values that were used to perform numerical simulations were obtained by inverse modelling methods applied to a measured bridge force waveform.⁶ It is the goal of the future work to assess bristle stiffness and damping from the measurements of rosin's properties and to incorporate measured Stribeck curves to the proposed thermal elasto-plastic friction model.

5. CONCLUSION

A refined elasto-plastic model incorporating temperature changes of rosin is presented. The underlying steady-state displacement function was adapted for viscosity changes due to temperature. Friction force when plotted versus relative velocity exhibits hysteresis loops, qualitatively agreeing with reconstructed hysteresis loops obtained from experimentally measured waveforms. To achieve a good match with experimental results, extensive measurements of rosin properties and subsequently extracting Stribeck curves for different rosin temperatures are needed. This could lead to further model refinement in order to accurately simulate bowed-string waveforms and generate numerical Guettler diagrams.

ACKNOWLEDGMENTS

This research was funded in whole or in part by the Austrian Science Fund (FWF) [10.55776/P34852].

REFERENCES

- ¹ P.R. Cobbold and M.P.A. Jackson. Gum rosin (colophony): A suitable material for thermomechanical modelling of the lithosphere. *Tectonophysics*, 210(3):255–271, 1992.
- ² P. Dupont, V. Hayward, B. Armstrong, and F. Altpeter. Single state elasto-plastic friction models. *IEEE Trans. Autom. Control*, 47(5):787–792, 2002.
- ³ P. Galluzzo. *On the playability of stringed instruments*. PhD thesis, 2004.
- ⁴ P. Galluzzo, J. Woodhouse, and H. Mansour. Assessing friction laws for simulating bowed-string motion. *Acta Acustica united with Acustica*, 103(6):1080–1099, 2017.
- ⁵ K. Guettler. On the creation of the helmholtz motion in bowed strings. *Acta Acust. united Acust.*, 88(6):970 – 985, 2002.
- ⁶ E. Matusiak and V. Chatziioannou. Elasto-plastic friction modeling toward reconstructing measured bowed-string transients. *J. Acoust. Soc. Am.*, 156(2):1135–1147, 2024.
- ⁷ E. Matusiak, V. Chatziioannou, and M. van Walstijn. Numerical modelling of elasto-plastic friction in bow-string interaction with guaranteed passivity. *Frontiers in Signal Processing*, 5, 2025.
- ⁸ M. McIntyre and J. Woodhouse. On the fundamentals of bowed string dynamics. *Acta Acustica united with Acustica*, 43, 09 1979.

-
- ⁹ M. E. McIntyre, R. T. Schumacher, and J. Woodhouse. On the oscillations of musical instruments. *The Journal of the Acoustical Society of America*, 74(5):1325–1345, 1983.
- ¹⁰ R. Pitteroff and J. Woodhouse. Mechanics of the contact area between a violin bow and a string. Part I: Reflection and transmission behaviour. *Acta Acust. United Acust.*, 84:543–562, 1998.
- ¹¹ R. Pitteroff and J. Woodhouse. Mechanics of the contact area between a violin bow and a string. Part II: Simulating the bowed string. *Acta Acust. United Acust.*, 84:744–757, 1998.
- ¹² R. T. Schumacher, S. Garoff, and J. Woodhouse and. Probing the physics of slip–stick friction using a bowed string. *The Journal of Adhesion*, 81(7-8):723–750, 2005.
- ¹³ S. Serafin, F. Avanzini, and D. Rocchesso. Bowed string simulations using an elasto-plastic friction model. In *Proceedings of the Stockholm Music Acoustics Conference*, 2003.
- ¹⁴ J. Smith and J. Woodhouse. The tribology of rosin. *J. Mech. Phys. Solids*, 48:1633–1681, 1999.
- ¹⁵ S. Willemsen, S. Bilbao, and S. Serafin. Real-time implementation of an elasto-plastic friction model applied to stiff strings using finite-difference schemes. In *Proceedings of the International Conference on Digital Audio Effects*, pages 40–46, 2019.
- ¹⁶ J. Woodhouse. Bowed string simulation using a thermal friction model. *Acta Acustica united with Acustica*, 89:355–368, 2003.
- ¹⁷ J. Woodhouse, R. T. Schumacher, and S. Garoff. Reconstruction of bowing point friction force in a bowed string. *The Journal of the Acoustical Society of America*, 108(1):357–368, 2000.